EST I – Math

Student's Name _____

National ID _____

Test Center _____

Duration: 90 minutes

Test sections: I- Calculator is not required, II – Calculator is required

45 Multiple Choice Questions and 13 Short Constructive Response Questions

Instructions:

- Place your answer on the answer sheet. Mark only one answer for each of the multiple choice questions.
- Write your final result only on the answer sheet for the constructive response questions.
- Avoid guessing. Your answers should reflect your overall understanding of the subject matter.
- Calculator is allowed. When a calculator is used, be aware of switching between radian mode and median mode.
- Formula sheet is available at the end of the booklet for your reference.

Section I
Calculator is not required
(30 minutes)

1. If $\frac{2x-4}{3}-\frac{x+1}{6}=t+1$ and $t = 3$, what is the value of x?

A. 27
B. -1
C. 11
D. $\frac{31}{3}$

2. $Z-Z'=a+bi$.

In the equation above, a and b are real numbers and i is the imaginary unit such that $i^2 = -1$.

If $Z = 3 + 2i$ and $Z' = 4 - 3i$, what is the value of $(Z - Z')^2$?

A. $2i$
B. $-24 - 10i$
C. $-26 - 10i$
D. $-1 + 5i$

3. Mathew paid X dollars for a play station that was only 30 dollars less than one third of the original price.

What was the original price in dollars?

A. $X-30$
B. $X+60$
C. $3X+90$
D. $\frac{1}{3}X-30$

4. A truck contains 15 identical boxes that are either red or blue.

The red box weighs 3 kg and the blue box weighs 2 kg.

If the total weight of the boxes is 36 kgs, what is the difference between the red and blue boxes in the truck?

A. 6
B. 9
C. 1
D. 3

5. A 100-page album costs twice as much as a 50-page album. The cost of three 100-page albums and two 50-page albums is $\$t$. How much does a 50-page album cost?

A. $8t$
B. $4t$
C. $\frac{t}{4}$
D. $\frac{t}{8}$

6. An enterprise conducted a study on its products and the results showed that when the unit selling price **(P)** is raised, the number of units sold **(U)** went down.

This result is modeled by the equation **10P+2U=2500.**

Based on this model, the manager decided to decrease the unit selling price from \$50 to \$45.

How many more items did he sell?

A. 75
B. 50
C. 25
D. 10

7. $\left[(2x-y)^2-(2x+y)^2\right]^2$

Which of the following is equivalent to the expression above?

A. $16x^4 - y^2$
B. $-64x^4y^4$
C. $-8x^2y^2$
D. $64x^2y^2$

8. If $\frac{3a}{b} \div c = 7$, what is the value of $\frac{bc}{2a}$?

A. $\frac{3}{14}$
B. $\frac{7}{3}$
C. 21
D. $\frac{6}{7}$

9. Which of the following is equivalent to $f(x) = 2x^2 - 12x + 8$?

A. $(2x-6)^2 - 28$
B. $2(x-3)^2 - 10$
C. $2(x-9)^2 + 5$
D. $4(x-3)^2 - 5$

10. $f(x) = ax^2 + (3-b)x - 5$

For the function f defined above, a and b are constants. If $f(1) = 2$ and $f(2) = -1$, which of the following is the value of $f(-1)$?

A. -22
B. -16
C. 2
D. 11

11. Sam wants to rent a car. He receives the following offers.

	Fixed amount to be paid	Amount to be paid for each kilometer driven
Offer A	65$	0.50$
Offer B	60$	0.52$
Offer C	55$	0.54$
Offer D	50$	0.56$

Which offer is the best if Sam wants to travel 100 km?

A. Offer A
B. Offer B
C. Offer C
D. Offer D

12. If m is a real parameter different than zero, what is the number of the real roots of the equation $2x(x^2 + 4)(mx^2 + x - m) = 0$?

A. 1
B. 2
C. 3
D. >3

13. $\sqrt{x^2+3} = x - 5$

Which of the following could be a solution for the equation above?

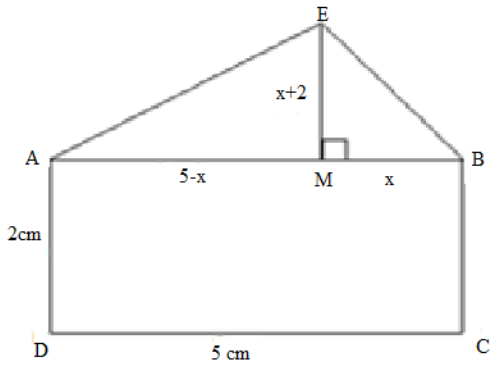
A. 0
B. 2.2
C. 5
D. None of the above

14. $25x^2 - tx + 4 = (5x-2)(ax+b)$

In the equation above, a, b and t are constant numbers.
What is the value of t?

A. 5
B. -2
C. 20
D. -15

15.

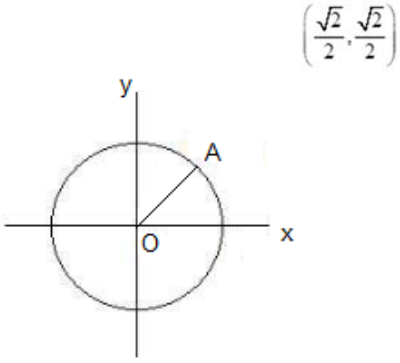


Knowing that $x < 5$, use the figure above to find the set of real numbers x if twice the area of triangle BME added to 2 times the area of the triangle MEA is less than or equal to triple the area of rectangle ABCD. (the figure is not drawn to scale)

A. $x < 4$
B. $x \leq 4$
C. $x \in [0,4]$
D. $x > 4$

16. If $3x - y = 1$ and $\frac{8^{2x}}{4^y} + t = 7$, what is the value of t?

17.



In the xy-plane above, O is the center of the circle and the measure of $\angle AOx$ is $\frac{4\pi}{a}$ radians. What is the value of a?

18.

x	$f(x)$
1	m
2	6
3	n

The table above shows some values for the function f.

If f is a linear function, what is the value of $m + n$?

19. $P(x) = (3a-6)x^2 + (4-2b)x + c - 3$

In the polynomial P above, a, b and c are constant numbers.

If P is identically zero, what is the value of $a + b + c$?

20. $\sqrt{x^2 - 5x + 8} = 2$

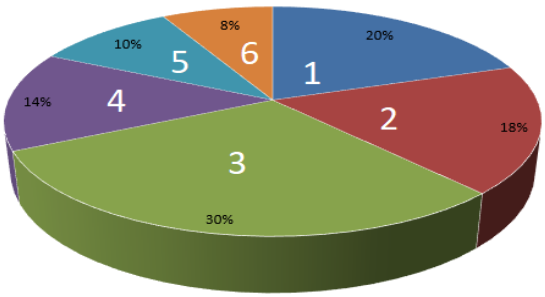
What is the product of the two solutions of the equation above?

Section II
Calculator is required
(55 minutes)

1. In 2017, country Y had 500 miles of paved roads. Starting in 2018, the country has been building 6 miles of new paved roads each year. At this rate, if m is the number of years after 2017, which of the following functions f gives the number of miles of paved roads that will be in country Y assuming that no paved roads go out of service?

A. $f(m) = 6 + 2017m$
B. $f(m) = 2017 + 6m$
C. $f(m) = 500 + 6m$
D. $f(m) = 2018 + 6m$

2.



The circular diagram above shows the results of a survey made on the number of books read by 200 pupils in a certain school.

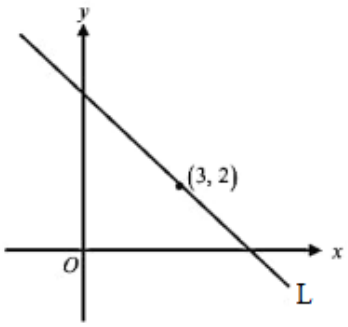
What is the number of pupils who read less than 3 books?

A. 68
B. 116
C. 38
D. 76

3. A machine finishes the paving of 600m of a road in 1 hour. At the same steady rate, how much time would take two identical machines to finish paving 300m?

A. 15 minutes
B. 2 hours
C. 1 hour
D. 30 minutes

4.



In the xy-plane above, the equation of line L is $2mx - 2y + 12 = 0$, where m is a constant.

What is the slope (gradient) of L?

A. $-\frac{4}{3}$
B. $\frac{2}{3}$
C. $-\frac{8}{3}$
D. $\frac{1}{2}$

5. Jack has k dollars. He spends $\frac{3}{4}$ of his money on a T-shirt and $\frac{1}{3}$ of what was left on a sandwich. If this left him with t dollars, which of the following is the value of k in terms of t?

A. $6t$
B. $9t$
C. $12t$
D. $24t$

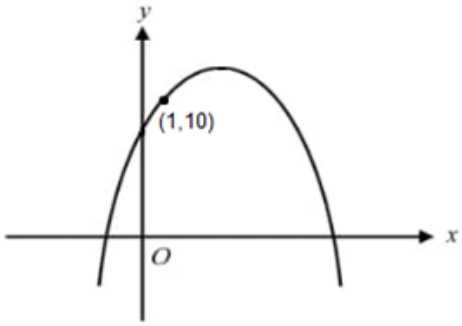
6. The supply function of a product is given by $f(x) = \frac{1+x}{2}$ and the demand function of the same product is given by $g(x) = \frac{2}{x} + 1$, where x represents the price in dollars of the product in both functions. What is the market equilibrium of this product given that it is the point of intersection of the two curves of the two functions?

A. 1.56
B. 1.79
C. 2.56
D. 2.79

7. Tina got on her exams the following grades:
Physics 75/100 with coefficient 2
English 60/100 with coefficient 4
Chemistry 80/100 with coefficient 1
Knowing that the coefficient of Math is 5, what should her minimum grade in Math be to have at least an average of 80/100?

A. 100
B. 85
C. 98
D. She won't be able to have at least this average

8.



Note: Figure not drawn to scale.

The graph of $y = tx^2 - 5tx - 6t$ is shown in the xy-plane above, where t is a constant. If the graph passes through the point (1,10), which of the following is the maximum value of y?

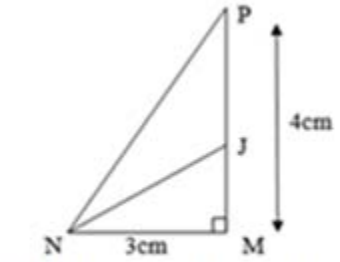
A. 2.5
B. 10
C. 11.75
D. 12.25

9. A study showed that the number of users of a mobile application has been growing exponentially with the number of new members doubling every 3 months. We know that the initial number of users when the study started was 100,000 users. The equation of the growth is given by $= 100e^{\alpha t}$ $(\alpha \in \mathbb{R})$, measured in thousands of users after t months.

How many users will the application have in 2 years?

A. 25,600 users
B. 25,600,000 users
C. 12,151 users
D. 12,151,041 users

10.



Note: Figure not drawn to scale.

Based on the figure above, what is the area of the triangle MNJ, if the area of the triangle MNJ is double the area of the triangle NJP?

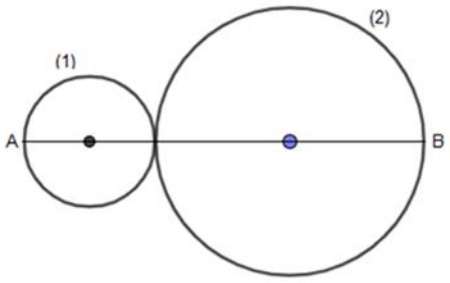
A. 2 cm^2
B. 3 cm^2
C. 4 cm^2
D. 5 cm^2

11. $f(x)=(x-2)(x+3)-(2x-4)^2$

Which of the following is an equivalent form of the function above?

A. $(x-2)(11+3x)$
B. $(x-2)(-x+7)$
C. $(x-2)(11-3x)$
D. $(x-2)(x-7)$

12.



In the figure above, the circles are tangent to each other. The radius of circle (1) is R and the radius of circle (2) is 2R.

If the sum of their areas is $100\pi\, cm^2$, what is the length of [AB]?

A. 100 cm
B. $2\sqrt{5}$ cm
C. 50 cm
D. $12\sqrt{5}$ cm

13. If $3x = 24y$, what is the value of $\left(\frac{3y}{x}\right)^2$?

A. $\frac{9}{64}$
B. $\frac{3}{4}$
C. $\frac{8}{3}$
D. 24

14. Tom has horses, dogs and birds in his farm. The number of birds he has is four times the number of dogs, and he has three more dogs than horses.

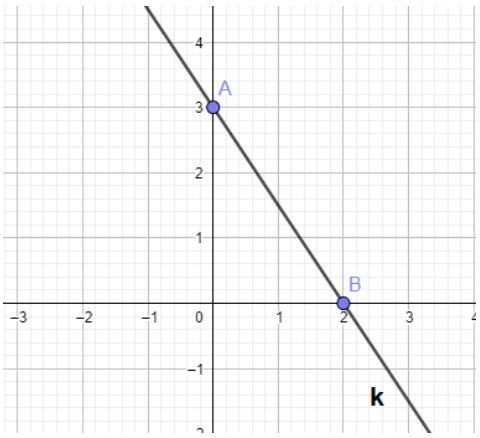
Which of the following could be the total number of these animals?

A. 25
B. 26
C. 27
D. 28

15. If x is different than -1, 0 and 1, which of the following is equivalent to $\frac{x}{\frac{1}{x-1}+\frac{1}{1+x}}$?

A. $\frac{1}{x^2-1}$
B. $\frac{(x-1)(x+1)}{2x}$
C. $\frac{2x}{x^2-1}$
D. $\frac{x^2-1}{2}$

16.



What is the equation of line **d** (not shown) that passes through origin O and is perpendicular to line **k** in the figure above?

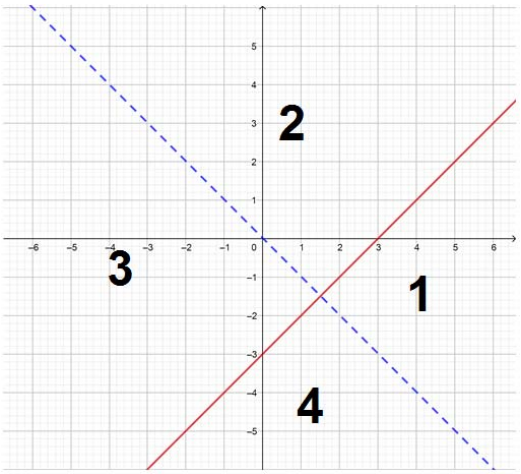
A. $y = \frac{2}{3}x$
B. $y = \frac{-3}{2}x$
C. $y = \frac{3}{2}x$
D. $y = x$

17. Line **m** in the xy-plane passes through the points $(2a, a^2)$ and $(2b, b^2)$ with $a \neq b$.

Which of the following is the slope (gradient) of line **m**?

A. $\frac{a}{b}$
B. $\frac{a+b}{2}$
C. $\frac{2}{a+b}$
D. $\frac{a^2-b^2}{a-b}$

18.



What is the solution region of the system $\begin{matrix} x - y \leq 3 \\ y > -x \end{matrix}$ represented in the graph above?

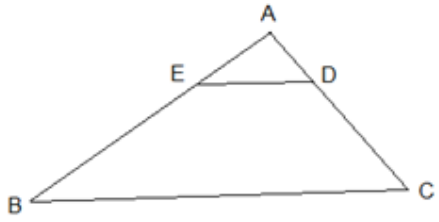
A. Region 1
B. Region 2
C. Region 3
D. Region 4

19. $\begin{cases} x + 3y = \frac{b}{2} \\ 4x + 4ay = 20 \end{cases}$

In the system of equations above, a and b are constants. If the system has one solution, which of the following could be the values of a and b?

A. $a = 3;\ b = 10$
B. $a = 3;\ b = 12$
C. $a = 3;\ b = -4$
D. $a = 10;\ b = 3$

20.



Note: Figure not drawn to scale.

In the figure above, $AE = \frac{1}{3}AB$ and $AD = \frac{1}{3}AC$.

The area of the triangle ABC is how many times the area of the trapezoid EDCB?

A. $\frac{9}{8}$
B. $\frac{8}{9}$
C. 3
D. $\frac{2}{3}$

21. A container is filled with 200 balls, 80 of them are yellow. After removing x yellow balls, 75% of the remaining balls in the container are not yellow.

Which of the following is the value of x?

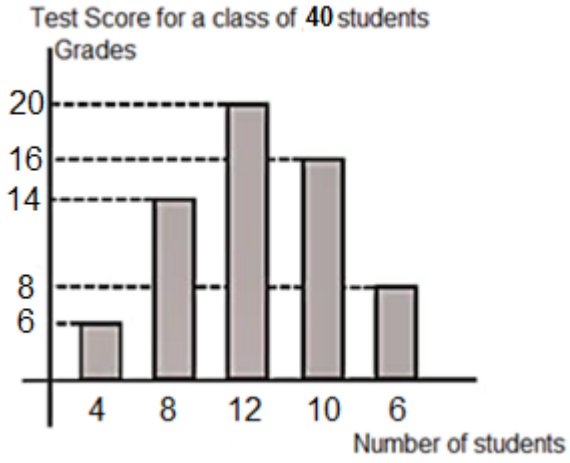
A. 20
B. 30
C. 40
D. 50

22. In 2007, a watch manufacturer found that 2 out of every 30 watches produced are defected.

If the manufacturer produces 2 million watches in a year, which of the following is closest to the estimated number of non-defected watches?

A. 1,866,000
B. 1,867,000
C. 133,000
D. 134,000

23.



The graph above shows the test grades over 20 of 40 students.

Based on the bar graph above, what is the average grade on the test?

A. 10
B. 11.6
C. 12.8
D. 14.6

24. For what value of x, $|4-x|-2$ is less than 0?

A. -7
B. 1
C. 3
D. There is no such value of x

25.

	Practice any kind of sports	No sports activity	Total
Under 40	220	40	260
40 and older	100	180	280
Total	320	220	540

The table above shows the distribution of age and sports activity for 540 employees of a company.

If an employee aged under 40 is selected at random, what is the probability that he practices any kind of sports?

A. $\frac{2}{13}$

B. $\frac{11}{27}$

C. $\frac{11}{16}$

D. $\frac{11}{13}$

26. Ryan estimates that there are x people in a concert.

Bella, who knows the actual number of people who attended the concert, y, notes that Ryan's estimate is within 50 people of the actual number of people.

Which of the following inequalities represent the relationship between x and y?

A. $|x-y| \le 50$

B. $x \le y+50$

C. $x \ge y-50$

D. $x+y \le 50$

Questions 27-28 refer to the following information.

$$f' = \left(\frac{v+v_0}{v}\right) f$$

You are riding in a car at a velocity v_0, in meters per second, towards a loud block party. Because of this movement, the actual frequency of the sound waves emitted by the speakers, f, in hertz, is perceived by you to be a different frequency f', in hertz.

The speaker's sound waves travel at a velocity v, in meters per second. This phenomenon is called the Doppler effect. The formula above shows the relationship between these variables.

27. Which of the following expresses the velocity of the car v_0 in terms of the other variables?

A. $v_0 = \frac{f'-f}{vf}$

B. $v_0 = \frac{f-f'}{f'}v$

C. $v_0 = \frac{f}{f'}v$

D. $v_0 = \frac{f'-f}{f}v$

28. If the velocity of the car is 22 m/s, the velocity of the sound waves of the speaker is 340 m/s and you perceive the frequency of the speaker's sound waves to be 500 Hz.

Which of the following is the closest to the actual frequency of the speaker's sound waves?

A. 469

B. 532

C. 470

D. 533

29. The table below summarizes students' work placement after graduation, based on the major.

	Work in their field of study	Work in another field	Don't work	Total
Engineering	17,065	10,593	3,867	31,525
Business	18,547	11,753	3,243	33,543
Law	20,372	1,438	542	22,352
Total	55,984	23,784	7,652	87,420

Given a person who works outside his field of study, which of the following is closest to the probability that he majored in business?

A. 0.35
B. 0.06
C. 0.5
D. 0.13

30. $x^2 + y^2 - 6x - 4y + 9 = 0$

The equation above is an equation of a circle. Which of the lines represented by the equations below is tangent to this circle?

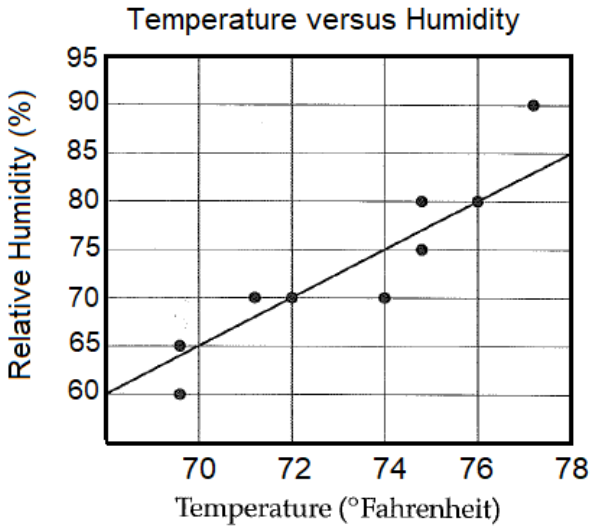
A. $y = \frac{1}{3}x - \frac{1}{3}$
B. $y = x$
C. $y = 3$
D. $x = 5$

31. A supermarket has three branches **A**, **B** and **C** in three different cities. The head manager realized that, in average per day, branch **A** has 20% more customers than branch **B** and branch **B** has 20% less customers than branch **C**.

If the number of customers in branch **A** is 1200 on a random day, what is the estimated number of customers in branch **C** that same day?

Questions 32-33 refer to the following information.

Humidity percentage vary according to the temperature. The scatterplot below compares the temperature, in degree Fahrenheit and relative humidity on a certain day, every hour from 12:00 P.M. to 8:00 P.M. The line of best fit is also shown.



32. Based on the line of best fit, what is the predicted humidity percentage at a temperature of 74° Fahrenheit?

33. What is the humidity percentage represented by the data point that is farthest from the line of best fit?

34. James adds to an oil tank 10 liters on day 1, then every day he adds 50% the amount he filled the day before.

After 1000 days, how much oil is there approximately in the tank?

35. Myriam opens a bank account with an initial deposit of 10,000 EGP. The bank account will earn 5 percent interest compounded annually for the first 3 years, after which it will earn 8 percent interest compounded annually.

What is the approximate amount, rounded to the nearest EGP, added to Myriam's account after 5 years?

36. A spinner has 2 blue sections, 3 red sections and 5 yellow sections. It is spun twice.

What is the probability of getting different colors?

Questions 37-38 refer to the following information.

Job title	Commission percentage rate
Trainee	1%
Employee	2%
Supervisor	3%
Senior supervisor	4%
Manager	5%

The chart above shows the commission structure for staff members working in a company.

All members of the staff benefit from a fixed salary of $800 monthly plus a commission on the profit of the company as shown in the chart.

37. The salary of the manager Sam for the month of January was $7,000. What is the salary of the supervisor John in January?

38. In February, the salary of an employee, a senior supervisor and a trainee was $13,650 altogether.

What was the approximate salary, rounded to the nearest dollar, of the trainee alone in February?