

1. NUMBERS

Even and Odd Numbers	
even+even=even	even-odd=odd
odd+odd=even	odd-even=odd
even+odd=odd	even·even=even
even-even=even	odd-odd=odd
odd-odd=even	even·odd=even

Positive and Negative Numbers	
positive+positive=positive	negative·negative=positive
negative+negative=negative	positive·negative=negative
positive-negative=positive	positive÷positive=positive
negative- positive =negative	negative+negative=positive
positive·positive=positive	negative÷positive=negative

Divisibility Rules	
2	the last digit of the number is 0, 2, 4, 6, or 8
3	the sum of the digits is divisible by 3
4	the last two digits are divisible by 4
5	the last digit is 0 or 5
6	the number is divisible by 2 and 3
9	the sum of the digits is divisible by 9
10	the last digit is 0

Squares	
$1^2 = 1$	$9^2 = 81$
$2^2 = 4$	$10^2 = 100$
$3^2 = 9$	$11^2 = 121$
$4^2 = 16$	$12^2 = 144$
$5^2 = 25$	$13^2 = 169$
$6^2 = 36$	$14^2 = 196$
$7^2 = 49$	$20^2 = 400$
$8^2 = 64$	$50^2 = 2500$

Exponents	
$2^1 = 2$	$3^3 = 27$
$2^2 = 4$	$4^3 = 64$
$2^3 = 8$	$5^3 = 125$
$2^4 = 16$	$3^4 = 81$
$2^5 = 32$	$10^3 = 1000$
$2^6 = 64$	$10^4 = 10000$
$2^7 = 128$	$(-2)^2 = 4$
$2^8 = 256$	$(-3)^3 = -27$

Prime Numbers
- A prime number is divisible by 1 and itself only
- The first 10 prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
- 1 is not a prime number
- 2 is the smallest and the only even prime number

v - speed t - time d - distance	$v = \frac{d}{t}, \quad d = vt, \quad t = \frac{d}{v}$
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Average Speed	$\frac{\text{total distance}}{\text{total time}}$
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%	Decimal	Fraction	%	Decimal	Fraction
1%	0.01	1/100	40%	0.40	2/5
2%	0.02	1/50	50%	0.50	1/2
5%	0.05	1/20	60%	0.60	3/5
10%	0.10	1/10	75%	0.75	3/4
20%	0.20	1/5	80%	0.80	4/5
25%	0.25	1/4	100%	1.00	1

Arithmetic Sequence	$a_{n+1} - a_n = d$ $a_n = a_1 + (n - 1)d$
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Geometric Sequence	$\frac{b_{n+1}}{b_n} = r$ $b_n = b_1 r^{n-1}$
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Percent	$1\% = \frac{\text{number}}{100}$
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Percent Proportion	$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$
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Percent Change	$\frac{\text{change}}{\text{original amount}} \cdot 100\%$
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2- SYSTEM OF LINEAR EQUATIONS

One solution

no solution

infinite

(x,y)

$$m_1 = m_2$$

$$R.H.S = R.H.S$$

* parallel lines [*have the same slope but different y - intercept*]

Ex. $y = mx + b_1$

$$y = mx + b_2 \rightarrow b_1 \neq b_2$$

* Two perpendicular lines [*the product of two slopes*]

Ex. $y = mx + b_1$, $y = \frac{-1}{m}x + b_2$

2- Relation between two lines:

Line 1: $a_1x + b_1y = C_1$

Line 2: $a_2x + b_2y = C_2$

One sol

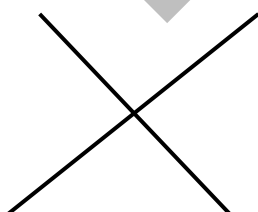
No sol

Infinite no.

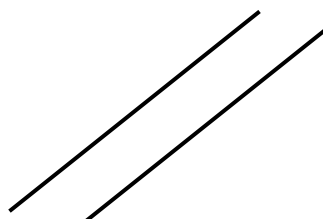
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

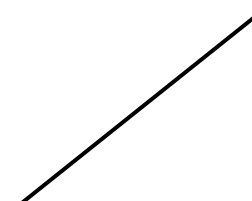
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$



Intersect

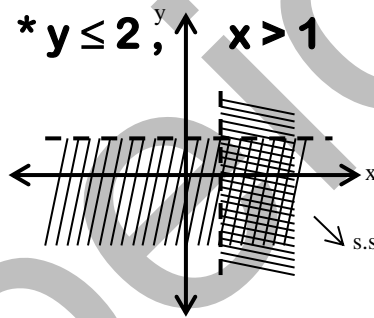
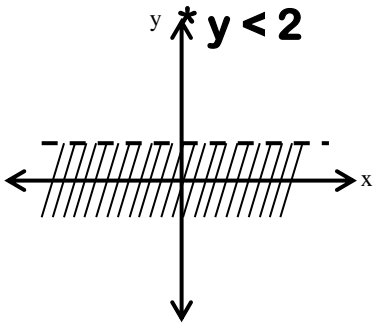
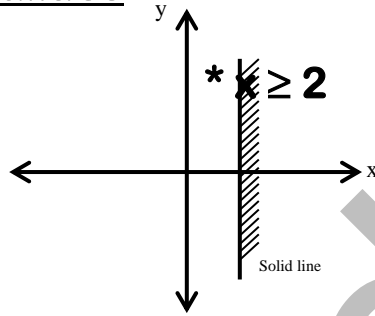
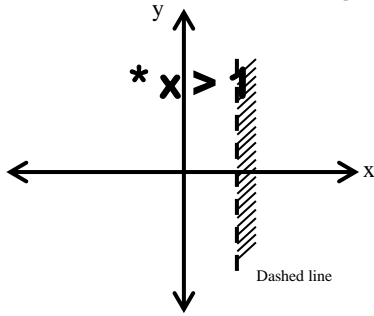


Parallel



Same line

3- Inequalities

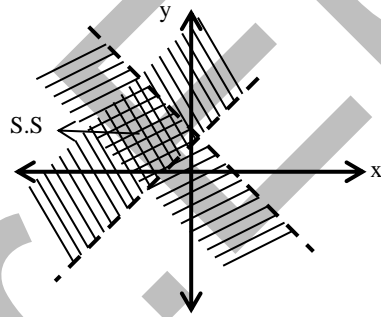


*** $y \geq 2x + 3$**

$y \geq 2x + 3$

$y - \text{int} = 3$

slope = 2



$y < -x + 1$

$y < -x + 1$

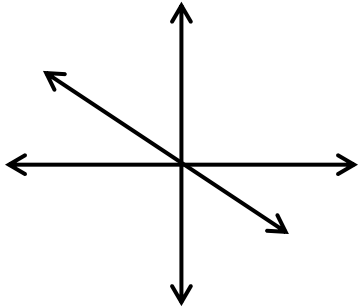
$y - \text{int} = 1$

slope = -1



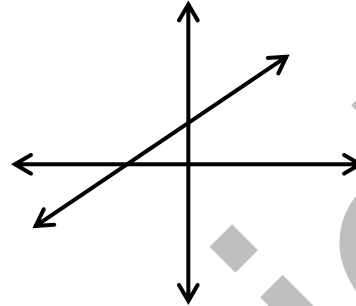
*** Negative slope**

Decreasing



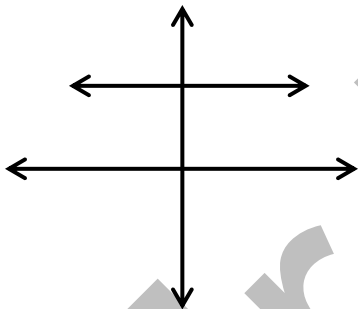
Positive slope

Increasing



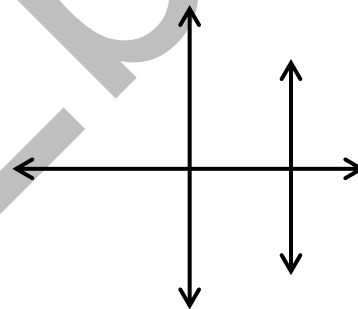
Zero slope

Constant – horizontal



Undefined slope

Vertical



QUADRATIC FORMULA AND DISCRIMINANT

$$ax^2 + bx + c = 0 \quad , \quad a \neq 0$$

Kind of roots

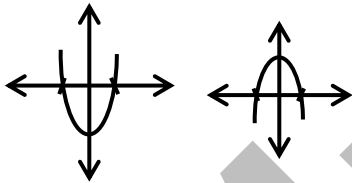
$$\Delta = \text{Discriminant} = b^2 - 4ac$$



$$\Delta > 0 \text{ (}\pm\text{ve)}$$

→ two diff.
roots(zeros-
solutions)

→ two x-intercept



$$\Delta = 0$$

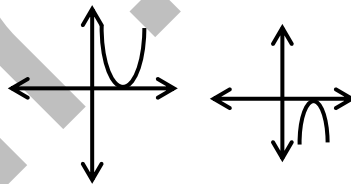
→ two equal real
sol.

→ double roots

→ one sol

→ one x-intercept

→ touch x-axis

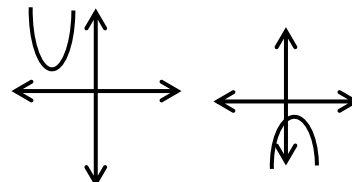


$$\Delta < 0 \text{ (-ve)}$$

→ no real sol

→ no x-intercept

Imaginary roots



Quadratic Equation

$$ax^2 + bx + c = 0$$

$$\text{Discriminant: } D = b^2 - 4ac$$

$b^2 - 4ac > 0$	two real solutions
$b^2 - 4ac = 0$	one real solution
$b^2 - 4ac < 0$	no real solution

Quadratic Formula

$$ax^2 + bx + c = 0$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solutions = Roots = X-intercepts=zeros

How to find the values of x in the quadratic function:

1 → Factoring () ()

2 → G. F = $\frac{-b \pm \sqrt{\Delta}}{2a}$

3 → **Mode** **5** **3** "Calc. Section"

Relation between two roots

$$ax^2 + bx + c = 0 \quad a \neq 0$$

$$\text{Sum} = \frac{-b}{a}$$

$$\text{Product} = \frac{c}{a}$$

6- QUADRATIC FUNCTION (PARABOLA)

1- General form $y = ax^2 + bx + c$

a: (+ve) \cup "Min. value"

a: (-ve) \cap "Max - value"

b: (+ve) shifted left

b: (-ve) shifted right

c: y- intercept at (o, c)

→ vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

→ axis of sym $x = \frac{-b}{2a}$

2- Vertex form $y = a(x-h)^2 + k$

a: $\begin{matrix} + & - \\ \cup & \cap \end{matrix}$

→ vertex (h, k).

h: $x = h$ axis of symmetry.

3- Factored form: "intercept form"

$y = a(x-m)(x-n)$

a: $\begin{matrix} + & - \\ \cup & \cap \end{matrix}$

→ x - intercepts $x = m, x = n$

→ axis of sym. $x = \frac{m+n}{2}$ → vertex $= \left(\frac{m+n}{2}, f\left(\frac{m+n}{2}\right)\right)$

→ y- intercept = amn

7- EXPONENTS – POWERS

Rules of Exponents		Rational Exponent	Radicals
$x^n = \underbrace{x \cdot x \cdot \dots \cdot x \cdot x}_{n \text{ times}}$ $x^a \cdot x^b = x^{a+b}$ $(x^a)^b = x^{ab}$ $(xy)^a = x^a \cdot y^a$	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$ $x^{-b} = \frac{1}{x^b}$ $\frac{x^a}{x^b} = x^{a-b}$	$x^{\frac{1}{n}} = \sqrt[n]{x}$ $(\sqrt[n]{x})^n = x$ $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ $x^{\frac{1}{2}} = \sqrt{x}$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ $\sqrt[n]{a^b} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $\sqrt{x^2} = x $ $a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x}$
Absolute Value		Direct Variation	Inverse Variation
$ 3 = -3 = 3$ $ x = -x $ $ x \geq 0$	$ xy = x y $ $\left \frac{x}{y}\right = \frac{ x }{ y }$	$\frac{y}{x} = k \text{ or } y = kx$	$xy = k \text{ or } y = \frac{k}{x}$

8- FACTOR THEOREM

* If $\frac{f(x)}{g(x)} = p(x) + \frac{k}{g(x)}$ **k is the remainder**

* if **k = 0** → no remainder → **g(x) is a factor for f(x)**

* if **(x-a)** is a factor of **f(x)** → then **f(a) = 0**

* if **x** is a factor of **f(x)** → **f(0) = 0**

* if **(x-b)** is not a factor of **f(x)** then **f(b) ≠ 0**

12- Sum and difference of two cubes

$$X^3 - y^3 = (x-y) (X^2+xy+y^2)$$

$$X^3 - y^3 = (x-y) (X^2+xy+y^2)$$

13- Difference of two squares

$$x^2 - y^2 = (x - y)(x + y)$$

$$x^4 - y^4 = (x^2 - y^2)(x^2 + y^2)$$

$$= (x - y)(x + y)(x^2 + y^2)$$

$$x^2 - a = (x - \sqrt{a})(x + \sqrt{a})$$

$$x^2 - 1/a = (x - \frac{1}{\sqrt{a}})(x + \frac{1}{\sqrt{a}})$$

$$x^2y^2 - m^2 = (xy - m)(xy + m)$$

$$x^2 - y^2 \neq (x - y)^2$$

↙ ↘

$$x^2 + 2xy + y^2$$

9- SETUP EQUATION AND INEQUALITY

$$x \text{ is } \rightarrow x =$$

$$x \text{ is 5 more than } y \rightarrow x = y + 5$$

$$x \text{ is 2 fewer than } y \rightarrow x = y - 2$$

$$x \text{ is 2 more than twice } y \rightarrow x = 2 + 2y$$

$$x \text{ less than } y \rightarrow x < y$$

$$x \text{ is at least } m \rightarrow x \geq m$$

$$x \text{ is at most } m \rightarrow x \leq m$$

$$x \text{ is no more than } m \rightarrow x \leq m$$

One – half $x \rightarrow \frac{1}{2}x$

One third $x \rightarrow \frac{1}{3}x$

square root of $x \rightarrow \sqrt{x}$

square of $x \rightarrow x^2$

square of the sum of x and $y \rightarrow (x + y)^2$

how many more \rightarrow subtraction

Product of x and $y \rightarrow x y$

Reciprocal of $x \rightarrow \frac{1}{x}$

10 - UNDEFINED FUNCTIONS OF VALUES:

* f(x)	$\frac{1}{x}$	$\frac{1}{x(x-1)}$	$\frac{x-1}{(x+2)(x-3)}$	$\frac{x+3}{x^2-1}$
undefined				
at	X = 0	X = 0, 1	X = -2, 3	X = ±1
So x must be	≠ 0	≠ 0, 1	≠ -2, 3	≠ ±1

* f(x) = $\sqrt{g(x)}$ → g(x) ≥ 0 to be defined

11-Rate → x direct variation with y $x \propto y \rightarrow \frac{x_1}{x_2} = \frac{y_1}{y_2}$

→ x inversely proportion $x \propto \frac{1}{y} \rightarrow \frac{x_1}{x_2} = \frac{y_2}{y_1}$

Or $x_1 y_1 = x_2 y_2$

→ If Hana do a job in t_1 , hr,

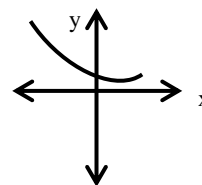
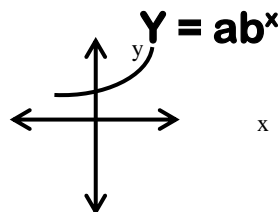
And Daren do a job in t_2 hr

If they work together it takes t_3 hr

s.t $\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{t_3}$

12- EXPONENTIAL FUNCTION AND RELATION

* $b > 1$ growth



* $0 < b < 1$ decay

→ to find b from the table → $b = \frac{y_2}{y_1} = \frac{y_3}{y_2} = \frac{y_4}{y_3}$

b: ratio, rate, double, triple



5. PROBABILITY AND STATISTICS

Probability of an Event	Odds
$\frac{\text{number of favorable outcomes}}{\text{number of all outcomes}}$	$\frac{\text{number of favorable outcomes}}{\text{number of unfavorable outcomes}}$

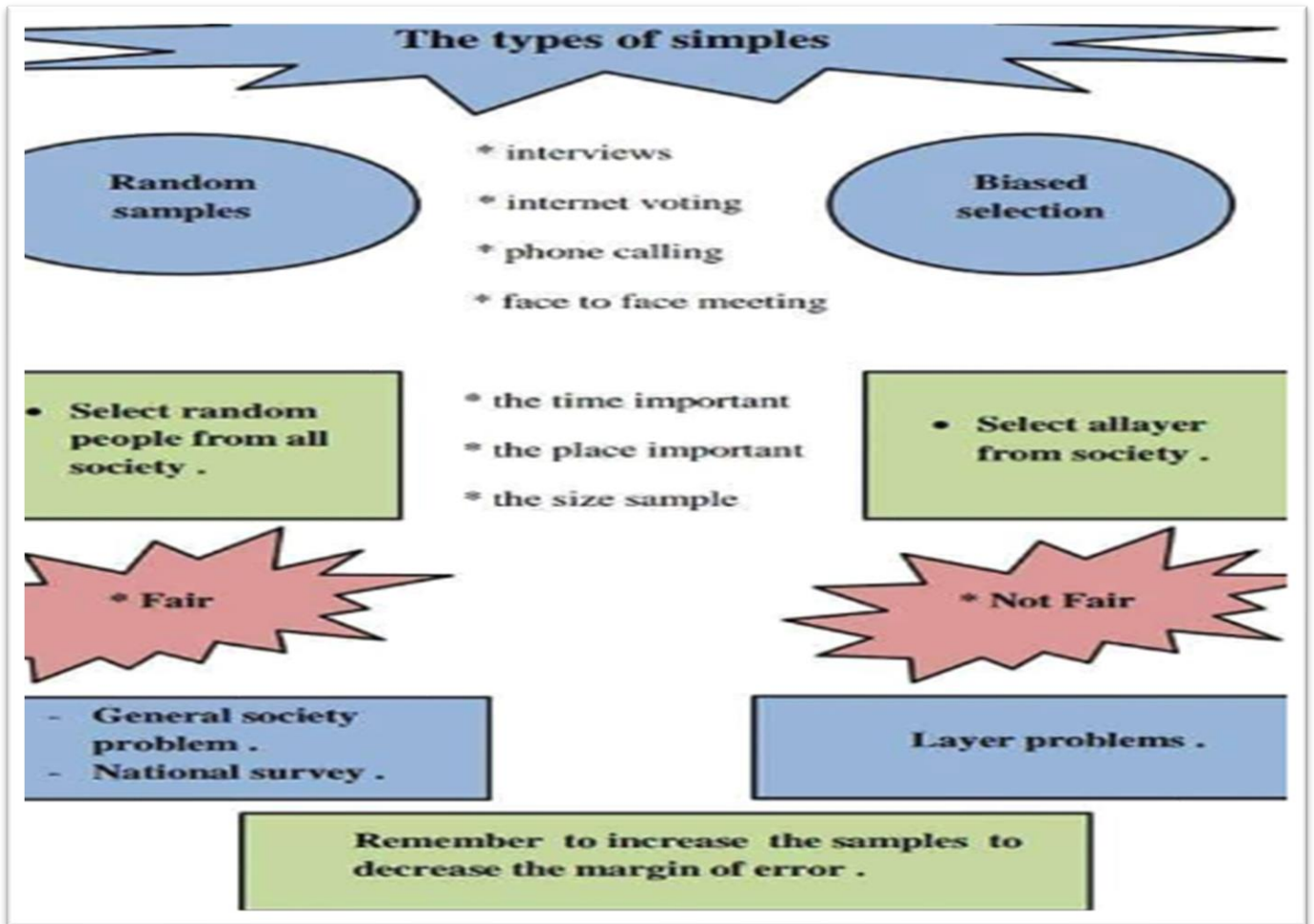
Independent Events	$P(A \text{ and } B) = P(A) \cdot P(B)$
Dependent Events	$P(A \text{ and } B) = P(A) \cdot P(B \text{ after } A)$

Geometric Probability	Fundamental Counting Principle
$\frac{\text{measure of favorable region}}{\text{measure of total region}}$	If one event can occur in M ways and another event can occur in N ways, then the total number of ways both events can occur is MN .

Mean	Factorial				
<p>The mean is the sum of the numbers divided by the number of values in the set.</p> $\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$ $x_1 + x_2 + x_3 + \dots + x_n = \bar{x} \cdot n$	$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \dots \cdot (n - 1) \cdot n$				
	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr style="background-color: #d9ead3;"> <th style="width: 50%; padding: 5px;">Permutations</th> <td style="padding: 5px;">$nP_m = \frac{n!}{(n - m)!}$</td> </tr> <tr style="background-color: #d9ead3;"> <th style="padding: 5px;">Combinations</th> <td style="padding: 5px;">$nC_m = \frac{n!}{(n - m)! m!}$</td> </tr> </table>	Permutations	$nP_m = \frac{n!}{(n - m)!}$	Combinations	$nC_m = \frac{n!}{(n - m)! m!}$
Permutations	$nP_m = \frac{n!}{(n - m)!}$				
Combinations	$nC_m = \frac{n!}{(n - m)! m!}$				

Median	The median is the middle value in an ordered set of numbers.
Mode	The mode is the value that occurs most frequently in a set of numbers.

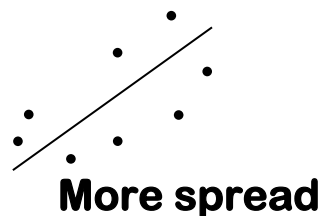
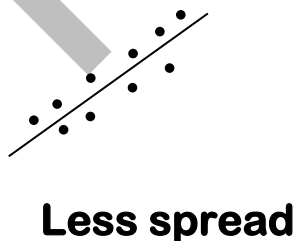
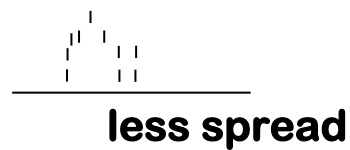
Mr. E



15- STANDARD DEVIATION

* less standard dev. Is more consistent

*



Small standard dev.

Great standard. dev.

Near to mean

Far from the mean

14- Margin error and confidence interval

If the true value is x and it has a margin error $r\%$

- so the confidence interval is $(x(1-r\%), x(1+r\%))$

Ex.. x has 5% margin error

So the confidence interval is between

$$X(1-0.05), x(1+0.05)$$

$$0.95x, 1.05x$$

Ex. Number of students came today 200

With more error 6%

$$200 \times (1-0.06) = 188$$

$$200 \times (1+0.06) = 212$$

>→ Con. Interval between them

16- Mode most repeated data (value)

17- Mean : (Average) = $\frac{\text{total}}{\#}$ (individual data) → x_1, x_2, x_3

$$\text{from table mean} = \frac{x_1f_1 + x_2f_2 + \dots}{f_1 + f_2 + \dots}$$

18- Median Arrange from least to greatest

* odd $\frac{n+1}{2}$ ← order (n. is odd)

* Even Two median s, their order $\frac{n}{2}$ (n: ever)

19 - Range Max. value - Min value

20- Ratio and compound of two ratios:

$$\frac{a}{b} = a:b$$

$$A : b = 2 : 3 \rightarrow a : b : \text{sum}$$

$$2 : 3 : 5$$

$$a = \frac{2}{5} \text{ of total}$$

$$b = \frac{3}{5} \text{ of total}$$

EX.)

$$a : b = 2 : 5 , \quad b : c = 5 : 2$$

$$a : b : c$$

$$2 : 5 :$$

$$: 5 : 2$$

$$10 : 25 : 10$$

$$\blacktriangle \text{ Percent of ratio } \frac{a}{b} = \frac{a}{b} \times 100\%$$

3. GEOMETRY (continued)

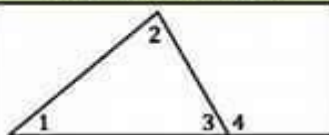
3

Inequalities in Triangles

Any side of a triangle is shorter than the sum of the two other sides

In a triangle, the longest side is opposite the largest angle

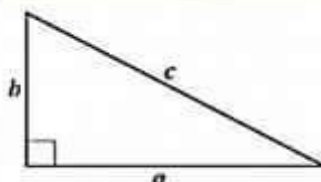
Angles of Triangles



$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

$$\angle 4 = \angle 1 + \angle 2$$

Pythagorean Theorem



$$c^2 = a^2 + b^2$$

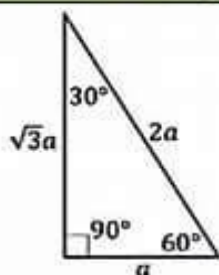
$$c = \sqrt{a^2 + b^2}$$

$$a = \sqrt{c^2 - b^2}$$

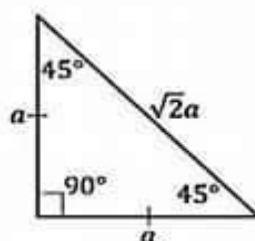
Pythagorean Triples

3, 4, 5
6, 8, 10
5, 12, 13
7, 24, 25
8, 15, 17
9, 12, 15

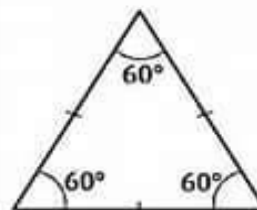
30°-60°-90° Triangles



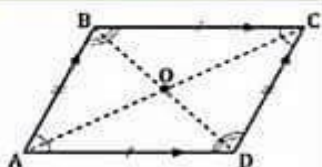
45°-45°-90° Triangles



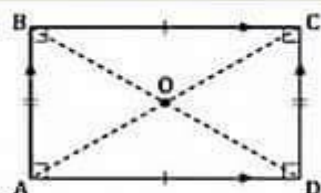
Equilateral Triangles



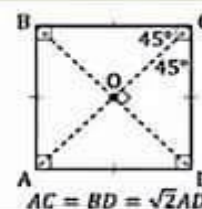
Parallelogram



Rectangle



Square



Area

Triangle	$\frac{1}{2}bh$
Parallelogram	bh
Rectangle	wl
Square	s^2
Trapezoid	$\frac{1}{2}(b_1 + b_2)h$

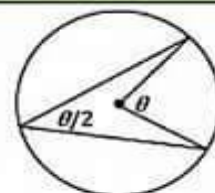
Perimeter

triangle	$a + b + c$
rectangle	$2(w + l)$
square	$4s$

Circle

Radius= r
Diameter= $2r$
Circumference= $2\pi r$
Area= πr^2

Circle



Central Angle= θ

Inscribed Angle = $\frac{\theta}{2}$

Arc length = $2\pi r \frac{\theta}{360}$

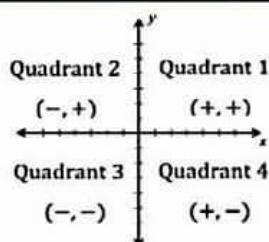
Solids

Solid	Volume	Surface Area
Rectangular Solid	$V = lwh$	$A = 2(lw + lh + wh)$
Cube	$V = s^3$	$A = 6s^2$
Right Cylinder	$V = \pi r^2 h$	$A = 2\pi r^2 + 2\pi rh$

4. COORDINATE GEOMETRY

4

The Coordinate Plane



Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

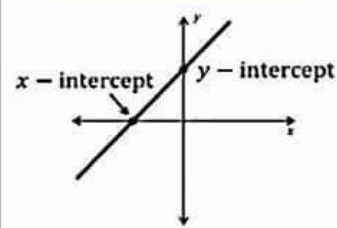
Slope-Intercept Form

$$y = mx + b$$

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Linear Functions

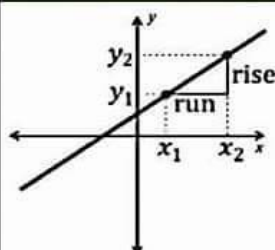


$$Ax + By = C$$

$$x\text{-intercept: } y = 0$$

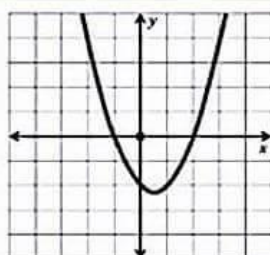
$$y\text{-intercept: } x = 0$$

Slope



$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Quadratic Functions



$$y = ax^2 + bx + c$$

$a > 0$ - parabola opens upwards

$a < 0$ - parabola opens downwards

Parallel lines

Parallel lines have the same slope

Perpendicular Lines

The product of the slopes of perpendicular lines is -1

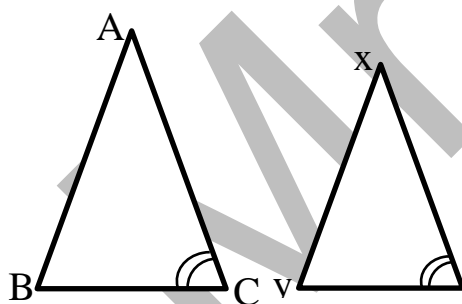
Vertical Shift

$y = f(x) + b$, up if $b > 0$, down if $b < 0$

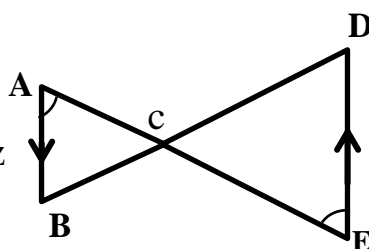
Horizontal Shift

$y = f(x + a)$, left if $a > 0$, right if $a < 0$

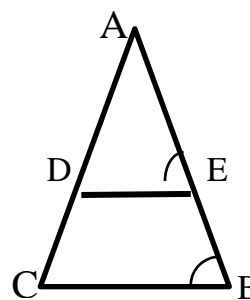
23- SIMILAR TRIANGLES



$$\triangle ABC \approx \triangle XYZ$$



$$\triangle ABC \approx \triangle EDC$$



$$\triangle ABC \approx \triangle AED$$

Then

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$$

$$\frac{AB}{ED} = \frac{BC}{DC} = \frac{AC}{EC}$$

$$\frac{AB}{AE} = \frac{BC}{ED} = \frac{AC}{AD}$$

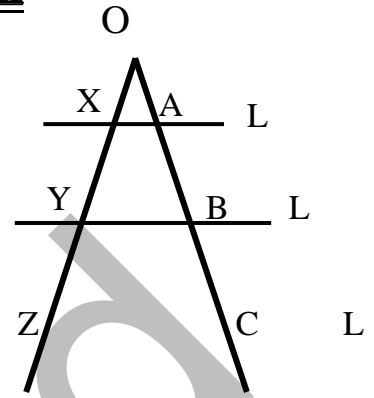
23- PARALLEL LINES IN TRIANGLE

$$L_1 \parallel L_2 \parallel L_3$$

$$1- \frac{OX}{OA} = \frac{XY}{AB} = \frac{YZ}{BC}$$

$$2- \frac{OA}{OB} = \frac{OX}{OY} = \frac{XA}{YB}$$

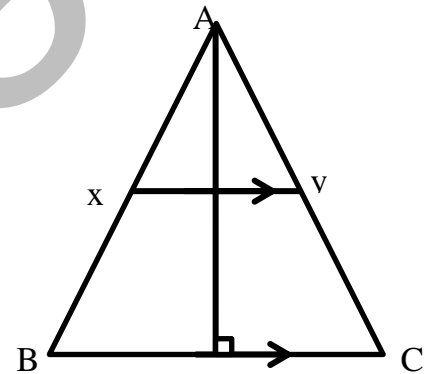
$$3- \frac{OY}{OZ} = \frac{OB}{OC} = \frac{YB}{ZC}$$



→ for the heights of Δ

$$XY \parallel BC$$

$$2- \frac{AX}{AB} = \frac{AY}{AC} = \frac{XY}{BC} = \frac{AN}{AM}$$



24- EQUATION OF THE CIRCLE:

* Standard form $(x-h)^2 + (y-k)^2 = r^2$

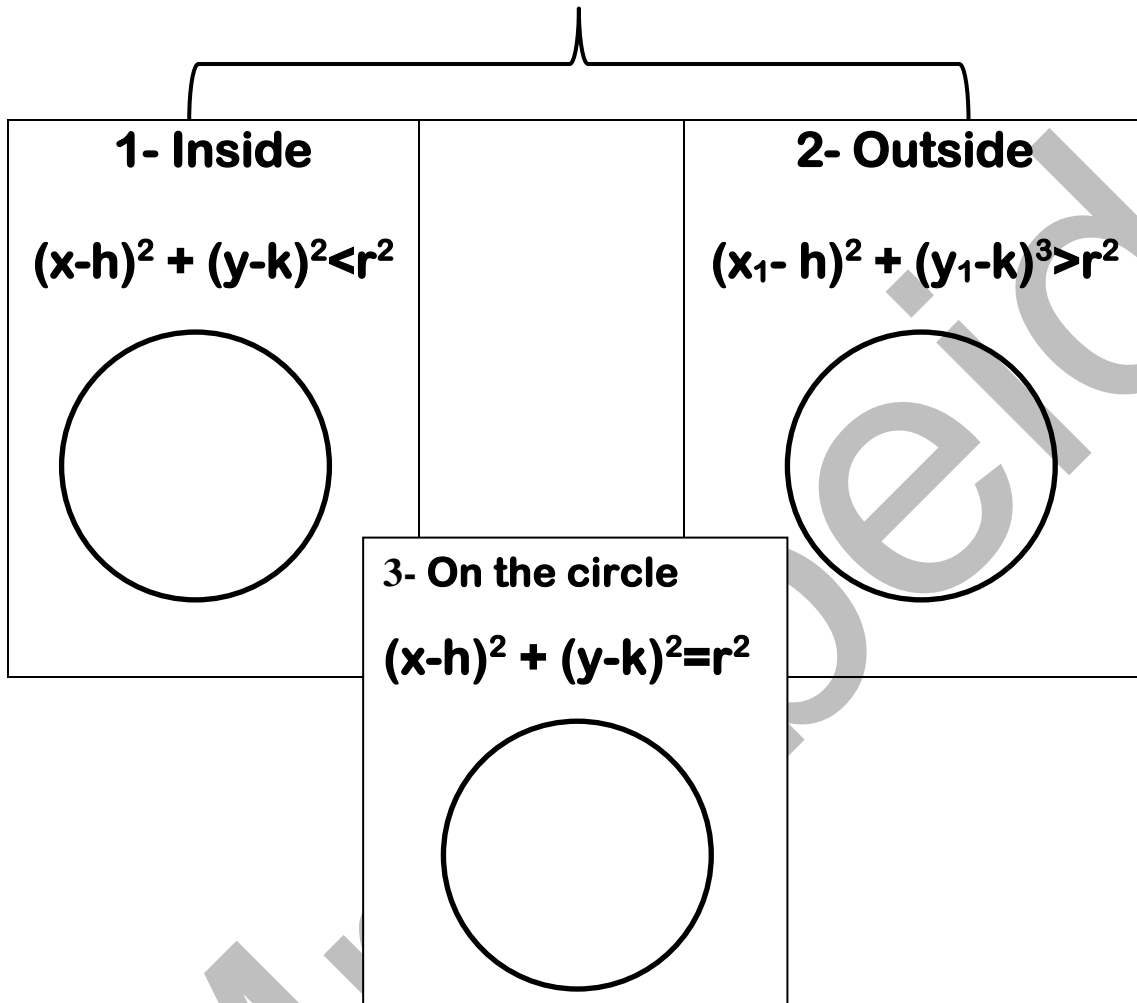
Centre: (h,k)

r : radius

* General form $x^2 + y^2 + ax + by + c = 0$

$$\text{Centre} = \left(\frac{-a}{2}, \frac{-b}{2} \right) \quad r = \sqrt{\left(\frac{a}{2} \right)^2 + \left(\frac{b}{2} \right)^2 - c}$$

Any Point (x_1, y_1)

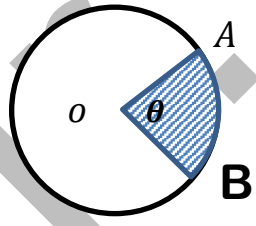
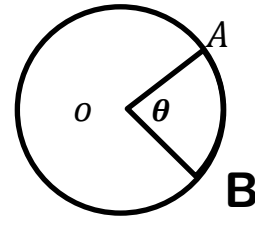


25- AREA OF SECTOR AND AREA LENGTH

$x^\circ = \text{in degree}$

$\theta^{rd} = \text{in radian}$

$$\frac{x^\circ}{\theta^{rd}} = \frac{180^\circ}{\pi}$$

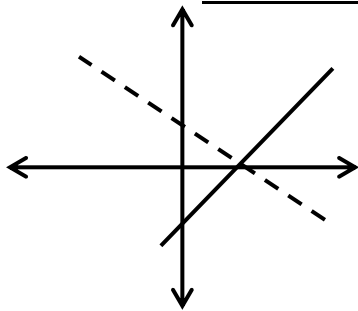
Area of sector	Length of the arc
$\frac{\theta^\circ}{360} \times \pi r^2$	$\frac{\theta^\circ}{360} \times 2\pi r$
	

* P of sector = Arc length + 2r

*Area of Sector in $\theta^{rd} = \frac{1}{2} r^2 \times \theta^{rd}$

*Arc Length in $\theta^{rad} = r \times \theta^{rad}$

26- REFLECTION IN X-AXIS AND Y-AXIS



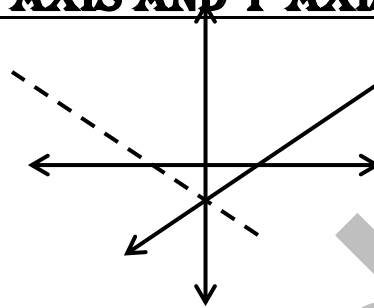
Ref. in X-axis

1- Opposite slope (-m)

2- Different Y-int

3- Same X - int

$$(x,y) \rightarrow (x, -y)$$



Ref. in Y-axis

1- Opposite slope (-m)

2- Same X- int

3- Different X -int

$$(x,y) \rightarrow (-x, y)$$

27- Transformation of graphs

$Y = f(x)$

Upward $\rightarrow f(x)+a$

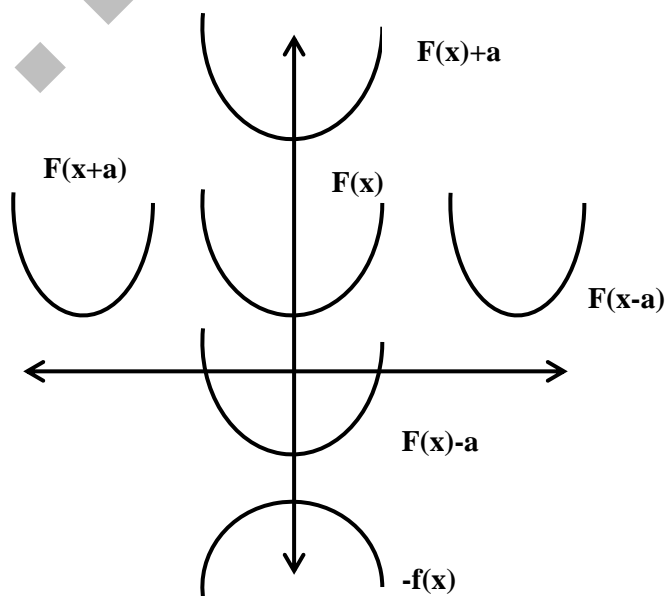
Downward $\rightarrow f(x)-a$

Shift left $\rightarrow f(x+a)$

Shift right $\rightarrow f(x-a)$

Ref in x $\rightarrow -f(x)$

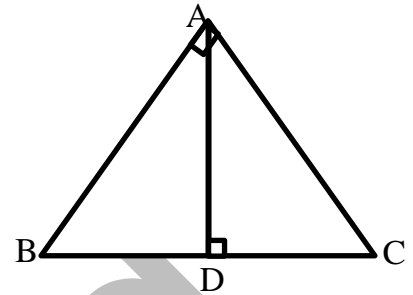
Ref in y $\rightarrow f(-x)$



28- Area of triangle

$$A = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times AB \times AC$$

Equilateral Δ $A = \frac{\sqrt{3}}{4} \times S^2$

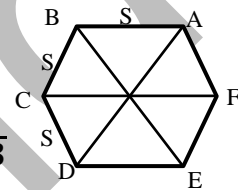


→ Hexagon

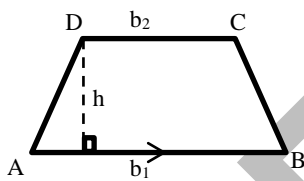
$$P = 6S$$

$$\text{diagonal} = 2S$$

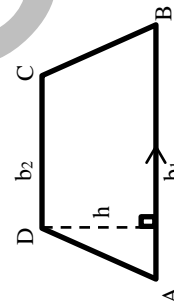
$$\text{Area } 6 \times \frac{\sqrt{3}}{4} S^2 \quad AC = CE = AE = \dots = S\sqrt{3}$$



29- Trapezoid



$$A = \left(\frac{b_1 + b_2}{2} \right) \times h$$

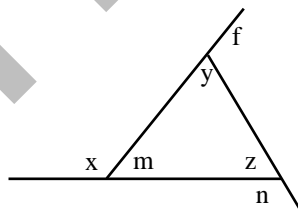


30- Exterior angles

$$X = y + z$$

$$F = m + z$$

$$N = m + y$$



* sum of exterior angles of any polygon = 360°

31- POLYGONS

* sum = $(n-2) \times 180$

n number of sides

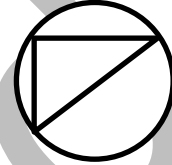
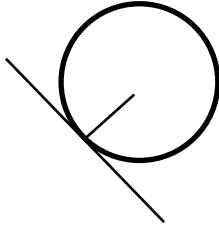
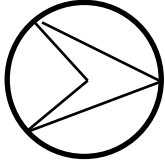
* Regular \rightarrow each angle = $\frac{(n-2) \times 180}{n}$

* Exterior angle = $\frac{360}{n}$

32- Inscribed and cent

$\rightarrow m(\angle ACB) = 90^\circ$

\rightarrow Tangent of circles is $\perp r$



33- Complex number $i = \sqrt{-1}$

$\sqrt{9} = 3$

$\sqrt{-9} = 3i$

$\sqrt{a^2} = a$

$\sqrt{-a^2} = ai \quad a > 0$

\rightarrow Complex number = $R + Img$

$Z = 5 + 7i$

$\rightarrow (a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i$

\rightarrow multiply

\rightarrow Division

Conjugate

$a + bi \rightarrow a - bi$

$i = \text{root } -1$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$